# Hollowing Out and Slowing Growth: the Role of Process Innovations*

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**Abstract**

I develop an endogenous growth model with skill acquisition, which can simultaneously account for labour market polarization and a slow down in labour productivity growth. When a new technology enters the economy, it requires implementation by high-skilled workers. Over time, process innovation makes the technology more user-friendly so that lower skilled workers can also operate it. Process innovation contributes to growth by increasing the range of technologies that a lower skilled worker can operate. It also reallocates labour demand for different skill-groups and thereby affects the income distribution. Skill can be acquired through a costly learning activity and workers face different learning costs. I calibrate the model to match the European labour market in 2000 and 2014 respectively. I show that when the rate of process innovation decreases, labour productivity growth slows down and wage and employment become polarized. (***JEL:*** O33, E24, J24)

***Keywords:*** labour market polarization, labour productivity growth, process innovation, product innovation, Europe

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# 1 Introduction

Labour market polarization is a widely observed phenomenon in a number of economies, both developed and emerging (see Reijnders and de Vries (2017) among many others). In particular, Goos, Manning and Salomons (2014) document that this phenomenon is pervasive in Europe over the period of 1993 to 2010. In these countries, the employment of low-paying and high-paying occupations has been growing faster than that of middle-paying occupations. Meanwhile, the wage of low-paying and high-paying occupations has also been growing faster than that of middle-paying occupations, resulting in the convergence of wages in the middle- and low-paying occupations.

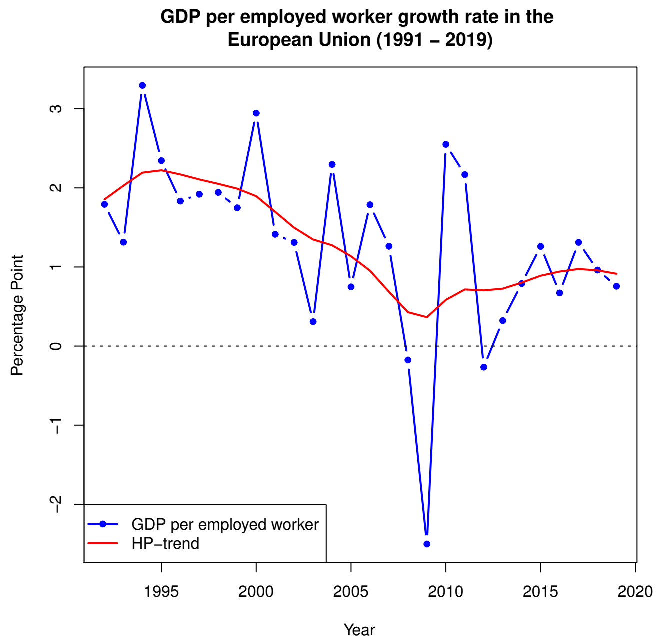
At the same time, labour productivity growth in Europe has been declining as well. Figure 1 plots the year-to-year growth rates for GDP per employed worker in the European Union from 1991 to 2019. The figure suggests that, in comparison to the 1990s, labour productivity growth declined substantially in the 2000s. For example, the trend rate of growth in GDP per worker was about 1.9% in 2000 and then dropped to about 0.8% in 2014 (see also the evidence in Adler et al. (2017); ECB Economic Bulletin (2017)).

The timing of these events motivates this paper to study job market polarization and slowing down labour productivity growth in Europe jointly in a single framework.[1] In particular, I develop a model which can generate both of the aforementioned phenomena simultaneously. In the model, new intermediate goods are the result of *product* innovations, and then pass through three consecutive production stages: the start-up stage, the routine stage, and the manual stage. The difference between these three production stages is that they have different levels of skill requirement for their implementation: the start-up production stage requires high-skilled workers, the routine production stage requires middle-skilled workers, and lastly, the manual production stage requires low-skilled workers.

All new intermediate goods enter the economy through the start-up stage, and the transitions to the other two subsequent stages are stochastically regulated by two independent *process* innovations. One process innovation regulates the transition from the start-up to the routine stage, and the other regulates the transition from the routine to the manual stage. Process innovations essentially simplify existing technologies and make them more accessible to workers with lower-skill levels. In this sense, the relative intensity of innovations shapes the demand for workers of each skill level in the model.

[1] Jaimovich and Siu (2020) document that, in the United States, episodes of job market polarization tend to concentrate in economic downturns. They also propose that the two phenomena can be related.

Figure 1: Labour productivity growth rate in Europe from 1991 to 2019




Data source: WorldBank Development Indicator database.

On the supply side, labour skills can be acquired through learning. In particular, any worker can pay a one-time fixed cost to acquire any skill level as s/he chooses. However, workers are endowed with different levels of innate ability, and for any skill level, it is more costly for a low ability individual to acquire than for a high ability one. The interaction between the innovations and the skill acquisition determines the income distribution and the labour productivity growth rate in equilibrium.

An important feature of this model is that process innovations emphasize the "deskilling" aspect of technological advancement. Well-known examples of this type of innovations include assembly lines and interchangeable parts (Acemoglu (2002)). Modern examples of process innovation have also been discussed in the literature (see, for example, Autor (2015)). Basically, process innovations often tend to break down complicated production procedures into smaller and more manageable pieces, which is helpful in reducing the minimum skill requirement associated with a job. The introduction of robots and automation, can often be thought of as a specific type of process innovation, which disentangle different parts of a job and allocate the skilled but routine part to machines and the lower-skilled residual to human workers.

To generate job market polarization and a labour productivity slow down jointly, the model requires a declining rate of process innovations. This result finds support in the data. In Section 2, I document a new stylized fact showing that, the rate of process innovations is gradually slowing down in Europe between 2000 and 2014.[2] The observed decline in the rate of process innovation is substantial. For example, the average rate of process innovation is about 23% in 2000 and only about 16% in 2014, implying approximately a 27% decline.

To quantify the change in the rate of process innovation, I calibrate the model to the labour market situation in Europe in 2000 and in 2014 separately. I divide all workers into high-skilled, middle-skilled, and low-skilled three groups, based on their occupations, following Goos, Manning and Salomons (2014). Even though I do not target *the change* in the rate of process innovation, I find that the calibrated rate of process innovation between the high-skilled and the middle-skilled occupations declined by about 71% between 2000 and 2014. In addition, the calibrated rate of process innovation between the middle-skilled and the low-skilled occupations also declined by about 3%. The calibration exercise also implies that the (untargeted) cost of product innovation increased by 27.5% during this period, and the (untargeted) learning cost of acquiring high skill relative to middle skill increased by about 26%.

To further investigate the dynamic relation between the rates of process innovations and the labour market situations, I conduct two quantitative exercises. In these exercises, I gradually reduce the two rates of process innovations respectively (i.e., an AR (1) shock), from the 2000 calibrated level to the 2014 calibrated level. I find that as the rate of process innovation between the high- and the middle-skilled decreases over time, the labour productivity growth rate decreases, and the labour market becomes polarized, both in terms of employment and wages. In contrast, a decline in the rate of process innovation between the middle- and the low-skilled alone could not generate such phenomena.

Related Literature. There is a recent empirical literature on job polarization. See Autor and Dorn (2013), Goos, Manning and Salomons (2014), Michaels, Natraj and Van Reenen (2014), Cortes (2016), and Reijnders and de Vries (2017) among others. The reason for polarization proposed in this literature requires the displacement of workers that mainly perform routine tasks (i.e., concentrated mostly in middle-paying occupations), and the complementarity between goods and services.[3] The labour demand for

[2] In the data I observe only one rate of process innovation instead of two.

[3] The "services" here refer to low-skilled in-person services like hair-cutting and food service and does

middle-paying occupations shrinks, pushing these workers to take on low skill service jobs, like waiters/waitresses and janitorial jobs.[4] In contrast, my paper provides a framework to generate polarization mainly using the routinization hypothesis (Autor, Levy and Murnane (2003)), and does not require the complementarity between goods and low-skilled in-person services.

Second, this paper complements a recent literature on automation, which emphasizes the labour saving aspect of technological improvements. For example, Acemoglu and Restrepo (2018) develop a model in which new tasks can be created, while existing tasks can also be automated. They assume that labour has a comparative advantage in new tasks. In the baseline model, there is only one type of labour, and they focus on the decline in labour share. In an extension of the model, they introduce high and low skill types of labour, and propose a similar stylized technological life-cycle, with high-skilled workers performing high-indexed (i.e., new) tasks, low-skilled workers performing middle-indexed tasks, and robots performing low-indexed tasks in equilibrium. They analyze how automation affects the skill premium in this scenario.[5] Another recent and related paper in the automation literature is Hémous and Olsen (forthcoming), in which they introduce automation into a Romer (1990) type growth model. Automation allows firms to replace low-skilled workers with machines. The decision about automation depends on the wages of low-skilled workers endogenously.[6]

A notable feature of my model is that high-skilled workers facilitate the transmission of new technologies (or labour demand) to the rest of the workforce, which relates to a couple of recent papers. In particular, Beaudry, Green and Sand (2018) provide empirical evidence supporting the critical role of entrepreneurial talent in the job creation process (see also Beaudry, Green and Sand (2016)). In addition, vom Lehn (2020) finds that adding a component of high-skilled workers being needed to adopt new technologies, would help substantially to bring the quantitative predictions of a standard routinization framework closer to the dynamics of labour market polarization in the United States.

Lastly, the environment and mechanics of my model shares similarities with Acemoglu, Gancia and Zilibotti (2012) and Lloyd-Ellis (1999). In particular, in the framework of Lloyd-Ellis (1999), there is a minimum skill requirement associated with each job and workers acquire skills exogenously to keep up with the growth in the skill requirement

not include skilled services like banking or medical care services.

[4]The model in Cortes, Jaimovich and Siu (2017) extends this framework by allowing middle-skilled workers to become unemployed.

[5]Service occupations are excluded from their framework.

[6]Other related papers also include Stokey (2018), Lee and Shin (2018), and Akcigit and Kerr (2018).

of jobs. In contrast, in my model jobs become more accessible over time as a result of process innovation, and workers acquire skills endogenously.[7]

The rest of the paper is organized as follows. Section 2 documents the declining rate of process innovation observed in Europe. Section 3 develops the model and establishes the equilibrium. Section 4 conducts the calibration exercise. Section 5 studies the transitional dynamics of declining rates of process innovation. Section 6 concludes.

## 2 Declining Process Innovation in European Countries

In this section, I document the declining rate of process innovations in Europe, using data from the Community Innovation Survey (CIS). The CIS is a firm level survey, which contains information about product and process innovations for most European countries from 2000 to 2016.[8] Each CIS survey covers firm's innovative activities for a three-year window before the survey year. For example, CIS2000 covers all the innovations from 1998 to 2000 inclusive; CIS2004 covers 2002 to 2004 inclusive, and so on. The data used in the paper are industry level aggregations, which is publicly available from the Eurostat website.

Process innovations occur in both service and manufacturing sector. To help firms to understand the definitions of various terminologies in the questionnaire and thereby to respond to the survey most accurately, the CIS provides a series of supporting documents. According to these documents, process innovations include new or improved production methods; logistics, delivery and distribution systems, and "back office" activities, such as maintenance, purchasing, and accounting operations. Notable examples of innovative methods of producing goods and services include installation of automation equipment or real-time sensors that can adjust processes. Examples of innovative logistics, delivery or distributional methods include introduction of passive radio frequency identification (RFID) chips to track materials through the supply chain.

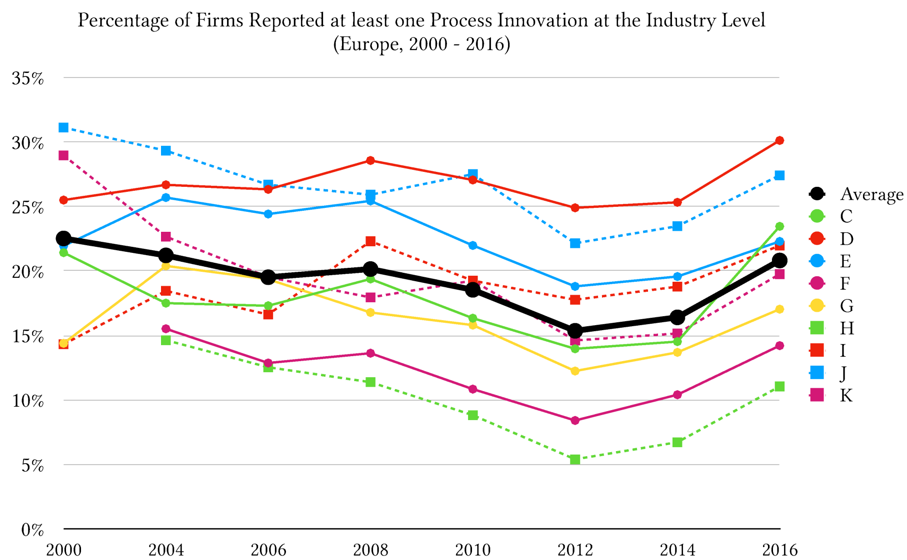
Note that if an enterprise has multiple process innovations during the survey period,

[7] Lloyd-Ellis and Roberts (2002) develop a model with minimum skill requirements and endogenous skill acquisition as well, but the learning setup is different.

[8] Starting with CIS 3, which was conducted in 2000/2001, a standard core questionnaire was developed and applied, in order to ensure comparability across countries. Therefore, my CIS data starts from 2000. Also, to avoid confusion, I use year as an indicator for each survey, as opposed to their ordinal numbers. For example, I refer CIS 3 as CIS2000. Therefore, I have CIS2000, 2004, 2006, 2008, 2010, 2012, 2014, and 2016, eight waves of surveys altogether.

the CIS only records it once.[9] As a result, for each industry, I observe the number of enterprises which reported *at least one* successful process innovation during the period under review. Figure 2 plots the percentage of these firms at the single digit industry level aggregated across countries. Although this statistic only captures part of the picture on the extensive margin, we observe that there is a gradual decline in the rate of process innovation in most industries between 2000 and 2014. The thicker black line denotes the average value of this statistic, which was about 23% in 2000 and only about 16% 2014. The decline in the rate of process innovation between 2000 and 2014 was approximately 27%.[10]

Figure 2: Declining Process Innovation in Europe




Data source: the Community Innovation Survey
Note (1): the industry coding is C - Mining and quarrying, D - Manufacturing, E - Electricity, gas, and water supply, F - Construction, G - Wholesale and retail trade; repair of motor vehicles, motorcycles and personal and household goods, H - Hotels and restaurants, I - Transport, storage and communication, J - Financial intermediation, K - Real estate, renting and business activities.
Note (2): In 2008, the "Statistical Classification of Economic Activities in the European Community" (i.e., the industrial classification) switched from Rev 1.1 to Rev 2. When consolidating the data set, I covert the Rev 2 activities to their Rev 1.1 equivalents based on the code book.

[9]On the questionnaire, it is a "yes/no" question regarding to situation about process innovations.
[10]Table 4 in the Appendix A provides more detailed information regarding Figure 2.

# 3 Process Innovation, Skill Acquisition, and Labour Market Polarization

In this section, I develop a model with product innovation, process innovation, and an endogenous skill distribution. I then discuss and define the competitive equilibrium of the model. I focus on solving the Balanced Growth Path (BGP) of the model. All the derivations are included in the Appendix B and C. I calibrate the model in Section 4 and discuss the transitional dynamics of the model in Section 5.

## 3.1 Production, Innovation, and the Demand for Skills

Time is continuous. There is a single final good in the economy, which is produced competitively using a large number of different intermediate goods. Intermediate goods are produced using labour inputs only. Depending on the stage of production, intermediate goods can be categorized into three groups: the start-up stage intermediate goods, the routine stage intermediate goods, and the manual stage intermediate goods.

When an intermediate good is invented, it enters the economy in the start-up stage. In this stage, the production technology associated with the good is assumed to be complicated, as it can be unfamiliar, and involves a lot of unconventional operations. As a result, the technology requires high-skilled workers to implement. More specifically, one unit output of start-up stage intermediate good requires one unit of labour input from high-skilled workers. *Skill* in the model can be thought of as a type of work qualification, and workers can acquire skills through a costly learning activity. The skill acquisition part is specified in Section 3.2.

Over time, production can, with some probability, move from the start-up stage to the routine stage. In the routine stage, the previously complicated production process has been broken down into a number of smaller and more manageable pieces, and the level of specialization between tasks has increased. As a result, one unit of output in the routine stage requires one unit of labour input from middle-skilled workers.

Eventually, production can stochastically move on to the manual stage, in which the production technology becomes even further simplified and one unit of intermediate output in this stage requires one unit of labour input from low-skilled workers.

Every new intermediate good enters the economy in the start-up stage, and the transition to other subsequent stages is a result of stochastic *process innovation*, which will be specified shortly.[11] Intermediate good producers compete with each other in a monopolistically competitive fashion in the intermediate goods market.

Intermediate goods enter the production of the final good according to the following constant elasticity of substitution (CES) production function,

$$Y(t) = N(t)^{\frac{2\alpha-1}{\alpha}} \left[ \int_{j\in N_L(t)} x_{L,j}^{\alpha}(t)dj + \int_{j\in N_M(t)} x_{M,j}^{\alpha}(t)dj + \int_{j\in N_H(t)} x_{H,j}^{\alpha}(t)dj \right]^{\frac{1}{\alpha}}, \alpha \in \left(\frac{1}{2}, 1\right). \tag{1}$$

In the production function, the three integrals denote the input from the three groups of intermediate goods, respectively. For example, in the first integral, $x_{L,j}(t)$ denotes the quantity of the manual stage intermediate good $j$ used in production of the final good at time $t$. Note that the manual stage of production has the lowest level of skill requirement and thereby the subscript $L$. Similarly, $N_L(t)$ denotes the measure of manual stage producers at time $t$. The second and the third integral are similarly defined for the routine stage (i.e., the subscript $M$) and the start-up stage (i.e., the subscript $H$) intermediate goods, respectively.

In addition, $\alpha$ is a measure of substitutability between different intermediate goods. The specification in Equation 1 effectively assumes that all the intermediate goods are equally productive in producing the final good, regardless of their individual stage of production. This is a simplification which can be relaxed. $N(t)$ is the measure of all intermediate good producers available at time $t$, therefore, $N(t) \equiv N_L(t) + N_M(t) + N_H(t)$, for all $t$.

The term $N(t)^{\frac{2\alpha-1}{\alpha}}$ introduces a positive aggregate externality that ensures the existence of a balanced growth path, whenever $\alpha \in (1/2, 1)$. By imposing Equation 1, I effectively assume a kind of knowledge spillover in the form of *learning-by-investing* in the finals good production sector: final goods producers that increase their use of intermediate inputs learn simultaneously how to produce more efficiently. So knowledge creation is a side product of investment (in the final goods sector). This setup is adopted from Acemoglu, Gancia and Zilibotti (2012).

Final goods producers choose intermediate inputs to minimize costs and earn zero

[11] Some professions may have never gone through this stylized life-cycle. It can be thought of as there are some stock intermediate professions in each stage at time zero.

profit, which yields the following demand functions for intermediate goods,[12]

$$x_{s,j}(t) = N(t)^{\frac{2\alpha-1}{1-\alpha}} \left(\frac{1}{p_{s,j}(t)}\right)^{\frac{1}{1-\alpha}} Y(t), \quad \forall s \in \{H, M, L\}, \text{ and } \alpha \in \left(\frac{1}{2}, 1\right), \tag{2}$$

where $p_{s,j}(t)$ denotes the price of intermediate good $j$ at stage $s$ at time $t$.

The transitions between different production stages occur due to process innovation, which are assumed to occur exogenously and follow a known Poisson process. More specifically, there are two independent Poisson processes in the model: one regulates the transition from the start-up stage to the routine stage, and the other regulates the transition from the routine stage to the manual stage. For any intermediate good, the first Poisson process applies immediately after the producer enters the economy, and the second Poisson process applies as soon as the production of the intermediate good enters the routine stage, if that ever occurs. The parameters for the first and second Poisson process are labeled as $\lambda_{HM}$ and $\lambda_{ML}$, respectively. Note that there is no innovation-related decision to make for intermediate producers after entry.

Given the structure of the process innovations, the probabilities that an intermediate producer, who entered at some time $t$, is operating in the start-up, the routine, or the manual stage at some time $v(> t)$ respectively, are

$$\begin{aligned}
\Phi_{H,t}(v) &= e^{-\lambda_{HM}(v-t)}, \\
\Phi_{M,t}(v) &= \left(1 - e^{-\lambda_{HM}(v-t)}\right) \frac{1 - e^{-\lambda_{ML}(v-t)}}{\lambda_{ML}(v-t)}, \text{ and} \\
\Phi_{L,t}(v) &= \left(1 - e^{-\lambda_{HM}(v-t)}\right) \left(1 - \frac{1 - e^{-\lambda_{ML}(v-t)}}{\lambda_{ML}(v-t)}\right).
\end{aligned}$$

In addition, the change in the measure of start-up stage producers at some time $t$, must equal the inflow due to product innovation, less the outflow to the routine stage due to the first process innovation. More specifically,

$$\dot{N}_H(t) = g(t)N(t) - \lambda_{HM} N_H(t), \tag{3}$$

where $g(t)$ denotes the rate of product innovation at time $t$. Similarly, for the changes in the measures of the routine and manual stage producers, respectively, we have

$$\dot{N}_M(t) = \lambda_{HM} N_H(t) - \lambda_{ML} N_M(t), \text{ and} \tag{4}$$

$$\dot{N}_L(t) = \lambda_{ML} N_M(t). \tag{5}$$

[12]The optimization problem for each agent and the first order conditions are formally set up and derived in the Appendix B.

Intermediate producers with the same level of skill requirements compete for workers, and wages are determined competitively in the labour market. Intermediate producers choose prices to maximize profits, which yields the following pricing rule:

$$p_s(t) = \frac{w_s(t)}{\alpha}, \quad \forall s \in \{H, M, L\}, \text{ and } \alpha \in \left(\frac{1}{2}, 1\right), \tag{6}$$

where $w_s(t), s \in \{H, M, L\}$ denotes the competitive wages received by each skill level of workers. It follows that intermediate producers at the same stage would choose the same price for their goods and supply the same quantity (see Equation 2). Accordingly, the corresponding profit levels can be expressed as

$$\pi_s(t) = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} w_s(t)^{-\frac{\alpha}{1-\alpha}} N(t)^{\frac{2\alpha-1}{1-\alpha}} Y(t), \quad \forall s \in \{H, M, L\}, \text{ and } \alpha \in \left(\frac{1}{2}, 1\right). \tag{7}$$

The arrival of new intermediate goods in the economy is a result of *product innovation*. There is free-entry associated with product innovation and innovators can incur a fixed cost $1/\eta$ and generate a new intermediate good (in the start-up stage). The fixed cost is in terms of the final good. Each intermediate good producer is assumed to receive a fully enforced perpetual patent on his/her variety. The present value for any product innovation enters at time $t$ is

$$\begin{aligned} V_H(t) = & \int_t^\infty \Phi_{H,t}(v)\pi_H(v)e^{-\bar{r}(t,v)(v-t)}dv \\ & + \int_t^\infty \Phi_{M,t}(v)\pi_M(v)e^{-\bar{r}(t,v)(v-t)}dv \\ & + \int_t^\infty \Phi_{L,t}(v)\pi_L(v)e^{-\bar{r}(t,v)(v-t)}dv, \end{aligned} \tag{8}$$

where $\bar{r}(t, v) \equiv \frac{1}{v-t}\int_t^v r(\omega)d\omega$, denotes the average interest rate between time $t$ and $v$. The free entry assumption will help to pin down the equilibrium.

## 3.2 Workers and the Supply of Skills

There is a measure $\mathcal{L}$ continuum of infinitely-lived, ex ante identical *households* in the economy. Each households is composed of an infinite stream of continuously overlapping generations of *workers*. Each individual worker remains active for $T$ units of time before exiting the economy permanently. At any given time, the size of the population is $\mathcal{L}T$. The size and the demographic composition of the population are constant over time.

When a new generation of workers enters the economy, each individual draws a *type* denoted by $\theta$, which is distributed uniformly between $\underline{\theta}$ and $\bar{\theta}$. This individual type can be thought of as a summary statistic of a person's innate ability and social economic background, including family wealth and connections, career preparation, physical health, intelligence and luck, as well as other factors which could affect how difficult it is for the individual to acquire skills.

If a type $\theta$ worker enters the economy at time $t$ and chooses to become low-skilled, then there is no cost, and the person can simply ignore his/her draw of $\theta$. Otherwise, the worker has to spend $\theta N(t)$ units of the final good if s/he chooses to acquire middle skill, or to spend $\mu\theta N(t)$ units if s/he chooses to acquire high skill. A lower $\theta$ should be thought of as "higher ability", as it costs *less* for a lower $\theta$ worker to acquire a given (i.e., high or middle) skill level than a higher $\theta$ worker. Additionally, the cost of skill acquisition is indexed to the total measure of intermediate goods, $N(t)$, which is for tractability reasons and to ensure the balanced growth path. Lastly, the parameter $\mu(>1)$ represents how much *more* expensive it is to acquire the high skill level than it is to acquire the middle skill level. There is no "on the job learning" after the initial skill acquisition.

Workers seek to maximize his/her contribution to the household's wealth, by acquiring the most rewarding skill level. This optimization behaviour leads to two cutoff levels in worker types, $\tilde{\theta}_{HM}(t)$ and $\tilde{\theta}_{ML}(t)$ such that

$$\int_t^{t+T} [w_H(v) - w_M(v)] e^{-\bar{r}(t,v)(v-t)} dv = \tilde{\theta}_{HM}(t) N(t) \cdot (\mu - 1), \text{ and} \tag{9}$$

$$\int_t^{t+T} [w_M(v) - w_L(v)] e^{-\bar{r}(t,v)(v-t)} dv = \tilde{\theta}_{ML}(t) N(t). \tag{10}$$

The left hand side of Equation 9 denotes the life-time income difference between the high-skilled and the middle-skilled, whereas the right hand side denotes the cost difference between acquiring high skill and middle skill for workers with type $\tilde{\theta}_{HM}(t)$. Equation 10 is specified in a similar fashion for the middle-skilled and the low-skilled and for workers with type $\tilde{\theta}_{ML}(t)$. Due to these two conditions, workers entering at time $t$ are partitioned into three groups, according to their draws of $\theta$. Specifically, workers with $\theta \in \left[\underline{\theta}, \tilde{\theta}_{HM}(t)\right]$ choose to acquire high skill, whereas workers with $\theta \in \left(\tilde{\theta}_{HM}(t), \tilde{\theta}_{ML}(t)\right]$ choose to acquire middle skill, and lastly, worker with $\theta \in \left(\tilde{\theta}_{ML}(t), \bar{\theta}\right]$ choose to acquire low skill.

Figure 3 provides a graphical illustration. The horizontal axis represents the worker type $\theta$ and the lower bound $\underline{\theta}$ is normalized to 0 in this case. The vertical axis represents the training cost associated with high and middle skill acquisition. The upward-sloping blue line denotes the cost to acquire the high skill level, whereas the upward-sloping black line denotes the cost to acquire the middle skill level. $\tilde{\theta}_{HM}(t)$ is determined when the cost difference between acquiring high skill and middle skill (i.e., the vertical distance between the blue line and the black line) equals the life-time discounted wage difference between a high-skilled worker and a middle-skilled worker (i.e., the LHS of Equation 9). Similarly, $\tilde{\theta}_{ML}(t)$ is determined when when the cost difference between acquiring middle skill and low skill (i.e., the vertical distance between the black line and the horizontal axis) equals the life-time discounted wage difference between a middle-skilled worker and a low-skilled worker (i.e., the LHS of Equation 10).[13]

Accordingly, I can represent the shares of the high-skilled, the middle-skilled, and the low-skilled workers enter at time $t$ (i.e., the skill distribution of "generation $t$"), with $\zeta_H(t)$, $\zeta_M(t)$, and $\zeta_L(t)$, respectively, which are defined as follows

$$\zeta_H(t) = \frac{\tilde{\theta}_{HM}(t) - \underline{\theta}}{\bar{\theta} - \underline{\theta}}, \tag{11}$$

$$\zeta_M(t) = \frac{\tilde{\theta}_{ML}(t) - \tilde{\theta}_{HM}(t)}{\bar{\theta} - \underline{\theta}}, \text{ and} \tag{12}$$

$$\zeta_L(t) = \frac{\bar{\theta} - \tilde{\theta}_{ML}(t)}{\bar{\theta} - \underline{\theta}}. \tag{13}$$

Note that $\zeta_H(t) + \zeta_M(t) + \zeta_L(t) = 1$ for all $t$.

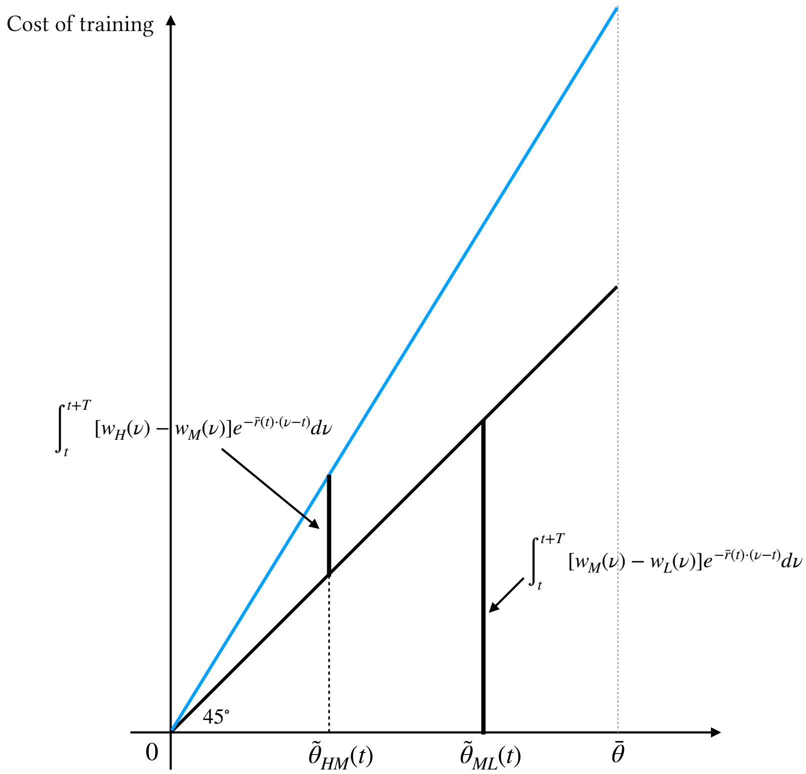
On the other hand, a representative household chooses a consumption plan to maximize utility, subject to an intertemporal budget constraint and a No-Ponzi game condition. The household's optimization behavior yields the following Euler equation,

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho, \tag{14}$$

where $c(t)$ denotes the household final good consumption at time $t$, $r(t)$ denotes the interest rate, and $\rho$ denotes the subjective discount factor.

[13]Note that I will need a condition under which $\tilde{\theta}_{HM}(t)$ is smaller than $\tilde{\theta}_{ML}(t)$, otherwise the equilibrium cannot exist. I am not able to derive this condition analytically, but I can check the values of $\tilde{\theta}_{HM}(t)$ and $\tilde{\theta}_{ML}(t)$ in the numerical exercise.

Figure 3: Supply of skills




Note: The upward-sloping blue line denotes the training cost for high skill, and the upward-sloping black line denotes the training cost for middle skill.

### 3.3 Labour Market Equilibrium and the Polarization Mechanism

The competitive labour market equilibrium implies that markets clear for all three skill levels

$$x_H(t)N_H(t) = \mathcal{L}\int_{t-T}^{t}\zeta_H(v)dv, \tag{15}$$

$$x_M(t)N_M(t) = \mathcal{L}\int_{t-T}^{t}\zeta_M(v)dv, \text{ and} \tag{16}$$

$$x_L(t)N_L(t) = \mathcal{L}\int_{t-T}^{t}\zeta_L(v)dv, \tag{17}$$

where the left hand sides are the demand for workers at each skill level at time $t$, and the right hand sides are the supply of workers at each skill level at time $t$. To simplify notations I define $\tau_H(t) \equiv \int_{t-T}^{t}\zeta_H(v)dv$, $\tau_M(t) \equiv \int_{t-T}^{t}\zeta_M(v)dv$, and $\tau_L(t) \equiv \int_{t-T}^{t}\zeta_L(v)dv$ for the rest of the paper.

In addition, denote the shares of the start-up, the routine, and the manual intermediate producers as $\chi_H(t)$, $\chi_M(t)$, and $\chi_L(t)$, respectively.[14] One can then derive the competitive wages for workers at each skill level, using Equations 2, 6 and the labour market clearing conditions 15, 16, and 17, as:

$$w_H(t) = \alpha y(t)^{1-\alpha}\left(\frac{\chi_H(t)}{\tau_H(t)}\right)^{1-\alpha} N(t), \tag{18}$$

$$w_M(t) = \alpha y(t)^{1-\alpha}\left(\frac{\chi_M(t)}{\tau_M(t)}\right)^{1-\alpha} N(t), \text{and} \tag{19}$$

$$w_L(t) = \alpha y(t)^{1-\alpha}\left(\frac{\chi_L(t)}{\tau_L(t)}\right)^{1-\alpha} N(t), \tag{20}$$

where $y(t) \equiv Y(t)/N(t)\mathcal{L}$, and

$$Y(t) = \left[\chi_L(t)^{1-\alpha}\tau_L(t)^{\alpha} + \chi_M(t)^{1-\alpha}\tau_M(t)^{\alpha} + \chi_H(t)^{1-\alpha}\tau_H(t)^{\alpha}\right]^{\frac{1}{\alpha}} N(t)\mathcal{L}. \tag{21}$$

To illustrate how this labour market can generate polarization, consider a decrease in the *relative* demand for middle-skilled workers. In this case, the relative wages for middle-skilled workers decreases, which implies that $w_H(t)/w_M(t)$ increases and $w_M(t)/w_L(t)$

[14] In other words, $\chi_H(t) \equiv N_H(t)/N(t)$, $\chi_M(t) \equiv N_M(t)/N(t)$, and $\chi_L(t) \equiv N_L(t)/N(t)$.

decreases. Consequently, when new workers enter the economy, those on the margin would find it worthwhile to acquire high skill and thereby $\tilde{\theta}_{HM}(t)$ moves to the right. Similarly, $\tilde{\theta}_{ML}(t)$ moves to the left as new workers on the margin find it not worthwhile to acquire middle skill. In other words, high ability worker would switch from middle skill to high skill, while low ability worker would switch from middle skill to low skill.[15] As new generations of workers adjust their skill acquisition behaviours, the stock of middle-skilled workers in the economy, $\tau_M(t)$ gradually decreases, whereas the stocks of both high- and low-skilled workers, $\tau_H(t)$ and $\tau_L(t)$ gradually increase. If the relative demand for middle-skilled workers keeps falling, then we observe a polarizing labour market in both wage and employment. The decrease in the relative demand for middle-skilled workers can be generated by changes in the rates of two process innovations, which will be discussed later in the quantitative exercise part.

Incidentally, given wages, the instantaneous profits for the start-up, the routine, and the manual intermediate producers can be expressed as

$$\pi_H(t) = (1-\alpha)y(t)^{1-\alpha}\left(\frac{\chi_H(t)}{\tau_H(t)}\right)^{-\alpha}\mathcal{L}, \tag{22}$$

$$\pi_M(t) = (1-\alpha)y(t)^{1-\alpha}\left(\frac{\chi_M(t)}{\tau_M(t)}\right)^{-\alpha}\mathcal{L}, \text{ and} \tag{23}$$

$$\pi_L(t) = (1-\alpha)y(t)^{1-\alpha}\left(\frac{\chi_L(t)}{\tau_L(t)}\right)^{-\alpha}\mathcal{L}. \tag{24}$$

## 3.4 General Equilibrium

The final good market equilibrium condition in the economy at time $t$ is

$$C(t) + I_{PD}(t) + I_L(t) = Y(t), \tag{25}$$

where $C(t)$ denotes the aggregate consumption, $I_{PD}(t)$ denotes the aggregate investment in product innovation, and $I_L(t)$ denotes the aggregate investment in skills acquisition.

[15]This result has a similar flavour to Cortes (2016), which documents that in the US, low ability routine workers are more likely to switch to nonroutine manual jobs, while high ability routine workers are more likely to switch to nonroutine cognitive jobs.

$I_{PD}(t)$ and $I_L(t)$ are defined as follows, respectively,

$$I_{PD}(t) = g(t)N(t)\frac{1}{\eta}, \text{ and} \tag{26}$$

$$I_L(t) = \int_{\theta \in \mathcal{H}(t)} \mu\theta N(t)d\theta + \int_{\theta \in \mathcal{M}(t)} \theta N(t)d\theta, \tag{27}$$

where $\mathcal{H}(t)$ denotes the set of workers choose to acquire high skill at time $t$, and $\mathcal{M}(t)$ denotes the set of workers choose to acquire middle skill at time $t$.

Given the parameters, a competitive equilibrium of the model consists of the following objects: aggregate output, aggregate investment, and aggregate consumption, $\{Y(t), I_{PD}(t), I_L(t), C(t)\}$; measures of intermediate good producers in the manual, the routine, and the start-up stages, $\{N_L(t), N_M(t), N_H(t)\}$; prices charged by intermediate good producers in different production stages, $\{p_L(t), p_M(t), p_H(t)\}$; profits received by intermediate producers, $\{\pi_L(t), \pi_M(t), \pi_H(t)\}$; wages for workers with low, middle, and high skill levels, $\{w_L(t), w_M(t), w_H(t)\}$, type cut-off levels for high skill and middle skill $\{\tilde{\theta}_{HM}(t), \tilde{\theta}_{ML}(t)\}$, and the interest rate $\{r(t)\}$, *such that*:

- Final good producers choose intermediate goods to minimize costs and earn zero profits (Equation 2).
- Intermediate good producers set prices and hire workers to maximize profits (Equation 6).
- Workers choose to acquire skills optimally according to their draws of $\theta$ (Equations 9 and 10).
- Households allocate their consumption and savings to maximize their utilities (Equation 14).
- Product innovators break even,

$$V_H(t) = \frac{1}{\eta}. \tag{28}$$

- The final good market, the intermediate goods market, the asset market, and the labour market all clear. In particular, asset market clearing implies that the following Bellman equations must hold for intermediate goods producers in each stage:

$$r(t)V_H(t) = \pi_H(t) + \lambda_{HM}[V_M(t) - V_H(t)] + \dot{V}_H(t), \tag{29}$$

$$r(t)V_M(t) = \pi_M(t) + \lambda_{ML}[V_L(t) - V_M(t)] + \dot{V}_M(t), \text{ and} \tag{30}$$

$$r(t)V_L(t) = \pi_L(t) + \dot{V}_L(t). \tag{31}$$

Note that $V_M(t)$ and $V_L(t)$ denote the values of an intermediate good producer in the routine stage and the manual stage, respectively. As usual, these equations require that the instantaneous profits from running a firm must equal the return from lending the market value of the firm at the risk-free rate. For example, in Equation 29, the return of lending the market value of an intermediate good producer at the start-up stage (i.e., the left hand side), must equal the instantaneous profits from operating at this stage plus, with probability $\lambda_{HM}$, the value change due to this producer transiting to the routine stage, and plus the value change due to time (i.e., the right hand side). A similar logic also applies to Equations 30 and 31.

## 3.5 Balanced Growth Path

There exists a Balanced Growth Path (BGP) in this economy, in which all aggregate variables grow at the same constant rate $\{\hat{g}\}$. Measures of intermediate producers at each production stage also grow at this rate, and thereby the shares of intermediate producers remain constant along the BGP. On the labour supply side, the skill acquisition type cutoffs $\{\hat{\theta}_{HM}, \hat{\theta}_{ML}\}$ are constant, so that while wages of workers at each skill level grow at the rate $\hat{g}$, the two skill premiums remain constant. Lastly, the interest rate $\{\hat{r}\}$ is also constant along the BGP.

Consequently, the BGP can be represented by (the steady state versions of) the free entry condition (Equation 32), the optimal skill acquiring conditions (Equation 33 and 34), and the Euler equation (Equation 35), which can be used to solve for the BGP equilibrium values of the growth rate, the two worker type cutoffs, and the interest rate, $\{\hat{g}, \hat{\theta}_{HM}, \hat{\theta}_{ML}, \hat{r}\}$:

$$\frac{\hat{\pi}_H}{\hat{r} + \lambda_{HM}} + \frac{\lambda_{HM}\left(\hat{\pi}_M + \lambda_{ML}\hat{\pi}_L/\hat{r}\right)}{[\hat{r} + \lambda_{HM}][\hat{r} + \lambda_{ML}]} = \frac{1}{\eta}, \tag{32}$$

$$(\hat{w}_H - \hat{w}_M)\,\frac{1 - e^{(\hat{g}-\hat{r})T}}{\hat{r} - \hat{g}} = \hat{\theta}_{HM}(\mu - 1), \tag{33}$$

$$(\hat{w}_M - \hat{w}_L)\,\frac{1 - e^{(\hat{g}-\hat{r})T}}{\hat{r} - \hat{g}} = \hat{\theta}_{ML}, \tag{34}$$

$$\hat{g} = \hat{r} - \rho. \tag{35}$$

Note that $\hat{\pi}_s = (1-\alpha)\hat{y}^{1-\alpha}(\hat{\chi}_s/\hat{\tau}_s)^{-\alpha}\mathcal{L}$ and that $\hat{w}_s = w_s(t)/N(t) = \alpha\hat{y}^{1-\alpha}(\hat{\chi}_s/\hat{\tau}_s)^{1-\alpha}$, for all $s \in \{H, M, L\}$, respectively. Derivations for the BGP equilibrium are included in the Appendix C.

**Proposition 1.** *If the Poisson rates of process innovation (i.e., $\lambda_{HM}$, and $\lambda_{ML}$) are large enough, then the model has a BGP equilibrium, which can be characterized by Equations 32, 33, 34, and 35.*

*Proof.* See Appendix C. □

# 4 Labour Market Polarization and Slowing Labour Productivity Growth

In this section, I calibrate the model to the European labour market in 2000 and 2014 separately. In doing so, I assume that the economy is in a BGP in 2000 and in a (potentially) different BGP in 2014. I discuss the model predictions by showing the trends of some untargeted moments are consistent with the data. In particular, the model predicts a decline in the rate of process innovation between 2000 and 2014, especially for the one between high-skilled and middle-skilled occupations. In the end, I also conduct a simple decomposition exercise and discuss the implication of each calibrated parameters and the mechanism of the model.

I first describe the calibration strategy. The parameter values used in the calibration exercise are presented in Table 2. A unit interval of time corresponds to one year. For externally determined parameters, I choose $\alpha = 0.8$, which implies the average markup level in the economy is 20%. This value matches the average markup in the Euro area between 1993 and 2004 estimated by Christopoulou and Vermeulen (2008). I set the working life-time $T = 45$ to approximate ages 20 to 64. According to the Worldbank Development Indicator Database, the average labour force in the European Union between 2000 and 2014 was approximately 207 million workers, together with the year of working life, I calculate the size of work force in each cohort $\mathcal{L} = 4.6$ (million). Lastly, I set the discount rate $\rho = 0.04$ and I normalize the lower bound of the worker skill type $\underline{\theta} = 0$.

The remaining five parameters of the model $\{\eta, \lambda_{HM}, \lambda_{ML}, \mu, \bar{\theta}\}$ are chosen to match five empirical targets that constitute a system of non-linear equations (i.e., Equations 32 to 35). The five empirical targets are the per worker GDP growth rate ($\hat{g}$), the skill premium of high-skilled relative to middle-skilled workers ($\hat{w}_H/\hat{w}_M$), the skill premium of middle-skilled relative to low-skilled workers ($\hat{w}_M/\hat{w}_L$), the proportion of high-skilled workers ($\hat{\tau}_H$), and the proportion of middle-skilled workers ($\hat{\tau}_M$).

While in the system the five parameters are jointly determined (together with all the other parameters in the model), each of them can most closely be linked to one specific target. For example, for (the inverse of) the cost of product innovation $\eta$, the key target is the annual GDP per worker growth rate. Additionally, for the Poisson rates $\{\lambda_{HM}, \lambda_{ML}\}$, the key targets are the two skill premiums, and for the learning cost of high skill relative to middle skill and the upper bound of the worker type $\{\mu, \bar{\theta}\}$, the key targets are the two labour skill proportions.

The annual GDP per worker growth rate is retrieved from the Worldbank Development Indicator Database. I apply the HP filter and the trend growth rate is used as the calibration target. The other four targets are obtained from the Structure of Earnings Survey (SES), which is a firm-level survey in European countries regarding wages and other characteristics of employees. The SES is conducted every four years starting in 2002 and there are 4 waves of SES available so far: 2002, 2006, 2010, and 2014.[16] The individual characteristics collected in the survey include age, gender, occupation, highest educational level achieved, and the length of service. The SES data used in this paper is the harmonized version and publicly accessible from the Eurostat website. More specifically, to obtain the calibration targets, I infer the information about labour supply and the wages, from the number of employees and the annual gross earnings from the SES, respectively.[17]

One important aspect of this exercise, is to empirically distinguish between high-, middle-, and low-skilled workers, for which I use occupation as the criterion, following Goos, Manning and Salomons (2014). Basically, occupations are ranked by their wages, and then separated into three groups. Here wages are used as a proxy for labour skills (see Autor and Dorn (2013)). More specifically, in my data, (1) Managers, (2) Professionals, and (3) Technicians and associate professionals are considered as high-skilled, (4) Clerical support workers, and (5) Plant and machine operators and assemblers are considered as middle-skilled, and lastly, (6) Service workers and shop and market sales

[16] The SES covers the 28 Member States of the European Union as well as candidate countries and countries of the European Free Trade Association (EFTA). The SES includes firms with at least 10 employees operating in all areas of the economy except public administration defined in the Statistical classification of economic activities in the European Community (NACE). The response rate varies across different countries and years. While the Eurostat quality report for SES 2014 is not available at the moment of writing, the average response rate is around 80% for SES2002, SES2006, and SES2010.

[17] I choose annual gross earnings over, for example, the hourly and monthly earnings, because annual earnings data "also includes allowances and bonuses which are not paid in each pay period, such as 13th month payments or holiday bonuses". I think these allowances and bonuses could be an important part of some high paying occupations, but sometimes are not reflected by the hourly or monthly earnings.

workers, and (7) Elementary occupations are considered as low-skilled.[18]

Table 1 presents the calibration targets in 2000 and 2014.[19] We can see that the labour market exhibits "polarization" in terms of both employment and wages. The skill premium of high-skilled workers (relative to the middle-skilled) increased by about 1.7%, and the employment share of such workers increased by about 9 percentage points. More interestingly, the skill premium of middle-skilled workers (relative to the low-skilled) decreased by about 8.1%, and the employment share of such workers also decreased by about 9 percentage points. The employment share of low-skilled workers barely changed during the same period. Lastly, the trend growth rate of GDP per employment worker in Europe decreased by more than a half between 2000 and 2014, from 1.9% to 0.8%.

Table 1: Calibration Targets in 2000 and 2014

| year | $\hat{g}$ | $\hat{w}_H/\hat{w}_M$ | $\hat{w}_M/\hat{w}_L$ | $\hat{\tau}_H$ | $\hat{\tau}_M$ |
|---|---|---|---|---|---|
| 2000 | 1.9% | 1.79 | 1.24 | 38% | 33% |
| 2014 | 0.8% | 1.82 | 1.14 | 47% | 24% |

Data source: The WorldBank Development Indicator Database and the Structure of Earnings Survey.

I calibrate the model to year 2000 and year 2014 respectively, assuming the externally determined parameters do not change. Table 2 summarizes the results of the calibration exercise. The model can hit all the calibration targets.

Comparing the calibrated parameters in 2000 and 2014 shows some interesting results. The implied rate of process innovation between high-skilled and middle-skilled occupations (i.e., $\lambda_{HM}$) decreased by about 71%, and the implied rate of process innovation between middle- and low-skilled occupations (i.e., $\lambda_{ML}$) also declined but to a much more moderate extent (about 3%). This result is consistent with the substantial decline in the percentage of firms that reported process innovations between 2000 and 2014 in the Community Innovation Survey. This result is useful as it shows that most of the reported decline in the process innovations are due to the decline in those reducing demand for high- relative to middle-skilled occupations, rather than those reducing demand for middle- relative to low-skilled occupations.

Intuitively, this result suggests that while fewer jobs/tasks were transmitted from

[18] I exclude "Skilled agricultural, forestry and fishery workers, craft and related trades workers" (OC6-7) and "Armed force occupations" (OC0).

[19] Note that I use the SES 2002 data as year 2000 data in the exercise here.

Table 2: Calibration Results

| Externally determined parameters | | | |
|---|---|---|---|
| Parameter | Value | | Source |
| Elasticity of substitution, $\alpha$ | 0.8 | | Christopoulou and Vermeulen (2008) |
| Years of working life-time, $T$ | 45 | | Ages 20 to 64 |
| Size of work force in each cohort (in millions), $\mathcal{L}$ | 4.6 | | The WorldBank Development Indicator Database |
| The discount rate $\rho$ | 0.04 | | Business cycle literature |
| The lower bound of the worker skill type, $\underline{\theta}$ | 0 | | Normalization |
| **Recovered using the calibration procedure** | | | |
| Parameter | 2000 value | 2014 value | Target |
| The inverse of the cost of product innovation, $\eta$ | 0.00255 | 0.00200 | GDP per worker trend growth rate, $\hat{g}$ |
| Poisson rate of PC from start-up to routine, $\lambda_{HM}$ | 0.00117 | 0.00034 | Skill premium of high- to middle-skilled, $\hat{w}_H/\hat{w}_M$ |
| Poisson rate of PC from routine to manual, $\lambda_{ML}$ | 0.00578 | 0.00560 | Skill premium of middle- to low-skilled, $\hat{w}_M/\hat{w}_L$ |
| Cost of high-skilled qualification rel. to middle, $\mu$ | 8.5 | 10.67 | Proportion of high-skilled workers, $\hat{\tau}_H$ |
| upper bound of worker's type, $\bar{\theta}$ | 2.9 | 1.83 | Proportion of middle-skilled workers, $\hat{\tau}_M$ |

middle-skilled to low-skilled occupations, *even fewer* were transmitted from high-skilled to middle-skilled occupations. As a result, the relative demand for middle-skilled workers decreased and wages became polarized. In response to this wage polarization, high-ability "middle-skilled" workers chose to become high-skilled, while low-ability "middle-skilled" workers chose to become low-skilled. Consequently, employment also became polarized.

The rest of the untargeted calibration results also make sense. In particular, the result implies that the cost of conducting product innovation (i.e., $1/\eta$) increased by 27.5%, which is consistent with findings in the literature (see Bloom et al. (2020)). Meanwhile, the cost of acquiring high skill relative to middle skill (i.e., $\mu$) also increased by about 26%. Lastly, the upper bound of worker's type $\bar{\theta}$ decreased by about 37%, which means that the average quality of workforce in Europe increased, and the per worker per skill level training cost has decreased.[20]

In order to isolate the impact of each parameter, I conduct a simple decomposition exercise. In this exercise, I change the parameter value one-by-one, from their 2000 value to their 2014 value (while keeping the rest of parameters at their value for the year 2000) and study the impacts on the targeted moments.[21] The results are shown in Table 3. The

[20] The exact quantity change in the average quality of workforce in Europe depends on the shape of the $\theta$ distribution.

[21] Note that if I reduce $\eta$, the inverse of the cost of product innovation, from its 2000 value to its 2014 value and keep the rest of the parameters at their year 2000 value, then the model does not have a solvable equilibrium, so I reduce $\eta$ as much as possible while the model maintains a sensible result. Consequently, in the decomposition exercise, $\eta = 0.00227$, instead of its 2014 calibrated level 0.00200. Due to the same

situations of year 2000 and 2014 are included as a reference in the top and the bottom row of the table respectively.

Table 3: Decomposition Exercise

| | $\hat{g}$ | $\hat{w}_H/\hat{w}_M$ | $\hat{w}_M/\hat{w}_L$ | $\hat{\tau}_H$ | $\hat{\tau}_M$ | $\hat{\tau}_L$ |
|---|---|---|---|---|---|---|
| 2000 | 1.9% | 1.79 | 1.24 | 38% | 33% | 29% |
| $\eta \Downarrow$ | 0.5% (−) | 1.49 (−) | 1.13 (−) | 30% (−) | 23% (−) | 46% (+) |
| $\lambda_{HM} \Downarrow$ | 2.0% (+) | 2.13 (+) | 1.31 (+) | 45% (+) | 26% (−) | 29% (+) |
| $\lambda_{ML} \Downarrow$ | 1.9% (+) | 1.79 (+) | 1.24 (+) | 39% (+) | 33% (+) | 28% (−) |
| $\mu \Uparrow$ | 0.8% (−) | 1.64 (−) | 1.15 (−) | 28% (−) | 28% (−) | 44% (+) |
| $\bar{\theta} \Downarrow$ | 3.4% (+) | 1.84 (+) | 1.22 (−) | 53% (+) | 32% (−) | 15% (−) |
| 2014 | 0.8% | 1.82 | 1.14 | 47% | 24% | 29% |

Data source: The WorldBank Development Indicator Database and the Structure of Earnings Survey.

First of all, I show the results of a reduction in $\eta$, which means an increase in the cost of conducting product innovation, in the second row of Table 3. The increase in cost stifles entry and brings the growth rate from 1.9% down to 0.5%. The slowing down in the entry of new products reduces the relative demand for high- and middle-skilled workers respectively, and thereby the skill premiums decrease. In particular, the skill premium of high relative to middle-skilled workers decreases from 1.79 to 1.49 and that of middle-relative to low-skilled workers decreases from 1.24 to 1.13. The declining skill premiums in turn reduce workers' incentive to acquire such skills. As a result, the overall labour force become less skilled in this case (i.e., both $\hat{\tau}_H$ and $\hat{\tau}_M$ decrease, while $\hat{\tau}_L$ increases).

Second, I show the results of a reduction in $\lambda_{HM}$, which means the rate of transmission of jobs/tasks from high-skilled occupations to middle-skilled occupations slows down. The results are shown in the third row of Table 3. This change first reduces the relative demand for middle-skilled workers, so that the skill premium of high-skilled workers increases, from 1.79 to 2.13. Meanwhile, there is now less incentive to become a middle skilled worker: incoming high ability "middle-skilled" workers would choose to acquire high skill, and incoming low ability "middle-skilled" workers would choose not to acquire middle skill and become low-skilled. As a result, employment becomes polarized (i.e., both $\hat{\tau}_H$ and $\hat{\tau}_L$ increase, while $\hat{\tau}_M$ decreases). In balance, the reduction in $\lambda_{HM}$ promotes a more skilled work force overall (i.e., most of the change in the work force is due to incoming high ability "middle-skilled" workers acquiring high skills). As a result, it encourages product innovation and thereby growth.

reason, in the decomposition exercise, $\bar{\theta} = 2.1$, instead of 1.83.

Third, I show the results of a reduction in $\lambda_{ML}$, which means a decrease in the rate of transmission of jobs/tasks from middle-skilled occupations to low-skilled occupations. The results are shown in the fourth row of Table 3. Similar to the analysis above, this change first reduces the relative demand for low-skilled workers, so that the skill premium of middle-skilled workers increases. As a result, incoming high ability "low-skilled" workers would choose to acquire middle skill. As a general equilibrium effect, the joining of high ability "low-skilled" workers put a downward pressure on the wage of middle-skilled workers, which in turn pushes high ability "middle-skilled" workers to acquire high skill. Consequently, this reduction in $\lambda_{ML}$ also promotes a more skilled work force overall, which induces product innovation and growth.

Fourth, I show the results of an increase in $\mu$, which means an increase in the cost of acquiring high skill relative to middle skill, in the fifth row of Table 3. This change has a similar effect to the increase in the cost of conducting product innovation, but in this case, the direct impact originates from the labour supply side. In particular, an increase in $\mu$ would directly discourage high skill acquisition, and through general equilibrium effects also discourage middle skill acquisition. As a result, the economy has a less skilled work force overall, which reduces the incentive of product innovation and growth.

Lastly, I show the results of a reduction in $\bar{\theta}$, which means a reduction in both the average and the variation of the per worker individual skill acquisition cost, in the sixth row of Table 3. This change induces more workers to acquire high skill and leads to a more skilled work force, which promotes entry.

## 5 Transitional Dynamics

In this section, I conduct two quantitative experiments, in which I gradually and permanently reduce the rate of process innovation, and study the transitional dynamics of the model. More specifically, I reduce the rates of process innovation $\lambda_{HM}$ and $\lambda_{ML}$ from their 2000 calibrated value to their 2014 calibrated value, respectively. The two experiments are similar in nature and thereby I describe the experiment on $\lambda_{HM}$ in detail and discuss the results of both experiments after.

To facilitate this exercise, I rewrite the model using discrete time. The timing assumption is as follows. At the beginning of period $t$, new intermediate producers and new workers enter the economy. All new intermediate producers would operate at the start-up stage, and new workers would receive their draw of $\theta$ and acquire skills accordingly. At the end of the period, intermediate producers would receive their profits, workers would receive their paychecks, and households would decide on consumption and investment (i.e., product innovation).[22] Lastly, and before entering period $t + 1$, process innovations occur, and start-up stage intermediate producers could stochastically enter the routine stage, and routine stage intermediate producers could stochastically enter the manual stage. Equilibrium conditions for the discrete time version of the model is analogous to those for the continuous time version and are provided in the Appendix D.

In the first experiment, the economy operates at the year 2000 BGP level starting from period 0. At the end of period 1, there is an unexpected AR (1) shock on $\lambda_{HM}$, which will gradually bring it down to its 2014 level. In particular, the AR (1) shock takes the following formation,

$$\lambda_{HM,t} = 0.3\lambda_{HM}^{ss} + 0.7\lambda_{HM,t-1}, \tag{36}$$

where the persistent parameter is 0.7. I change the value of $\lambda_{HM}^{ss}$ from its 2000 calibrated level to its 2014 calibrated level at the end of period 1. This shock implies that the transition from the start-up stage to the routine stage slows down.

Given the timing structure, the shock does not affect the choices of producers or workers in the economy at period 1. Intermediate producers and workers entering at the beginning of period 2, on the other hand, would have full information: they observe the current state of the economy, and they know the entire shock process. They will adjust their choices accordingly. The algorithm to calculate the transitional dynamics of the model is similar to a shooting algorithm and the details are provided in the Appendix D.[23]

Figure 4 presents the results of the first exercise. The top left panel shows the shock process, and the other eight panels show the implications. First of all, as the rate of process innovation $\lambda_{HM}$ decreases at the end of period 1, new workers entering at the beginning of period 2 would find that there is excessive demand for high-skilled workers, while middle-skilled workers are relatively oversupplied. Consequently, new workers on

[22]When calculating the present value life-time income, workers would discount the period $t$ income using the period $t$ interest rate $r_t$. Similarly, firms would discount the period $t$ profits using the interest rate $r_t$.

[23]Briefly, I first compute the two BGPs before and after the shock. I then guess policies for a sufficiently long period of time. The number of period has to be large enough so that the economy could reach the new BGP afterwards. Based on these guessed policies, I calculate a series of "future" states of the model, from the stand point of the period in which the shock occurs. I update the guesses until they are consistent with their implied future states.

the margin (i.e., the high ability "middle-skilled" workers) find it worthwhile to acquire high skill and $\tilde{\theta}_{HM}$ jumps up in period 2 (top middle). Meanwhile, new workers on the other margin (i.e., the low ability "middle-skilled" workers) find it *not* worthwhile to acquire middle skill and $\tilde{\theta}_{ML}$ drops down in period 2 (top right).

As a result, the shares of high-skilled workers and low-skilled workers increase (bottom left and bottom right), and the share of middle-skilled workers decreases (bottom middle). In the meantime, the skill premium for high-skilled workers increases (middle middle), and, due to general equilibrium effects, skill premium for middle-skilled workers also increases a tiny bit (middle right).

On the other hand, from the product innovator's perspective, now the start-up stage labour market is "tighter" than expected and the wage for high-skilled workers increases. Moreover, it is also less likely to process innovate and reach the routine stage. Therefore, product innovation slows down in period 2 (middle left).

As $\lambda_{HM}$ continues to fall, the rate of product innovation also keeps falling, and more incoming workers acquire high skill and low skill respectively. As long as the demand for high-skilled workers increases faster than the supply of high-skilled workers, both the employment share (bottom left) and the relative wage of high-skilled workers (middle middle) keeps increasing over time. Due to a similar reason, both the employment share (bottom middle) and the relative wage of middle-skilled workers (middle right) keeps decreasing over time. In sum, this exercise demonstrates that, as the rate of process innovation between the start-up stage and the routine stage gradually decreases, in the short run, the rate of labour productivity growth decreases and the labour market becomes increasingly polarized.

In the second exercise, I consider a similar negative shock on $\lambda_{ML}$, which implies that the transition from the routine stage to the manual stage slows down. More specifically, I consider a similar AR(1) process as in Equation 36 with the same persistent parameter. I reduce $\lambda_{ML}^{ss}$ from its 2000 calibrated level to its 2014 calibrated level at the end of period 1, assuming again the economy operates at the 2000 BGP level in period 0. The top left panel of Figure 5 shows the shock process, and the rest show the implications.

The direct impact of this shock increases the relative demand for middle-skilled workers so that new workers on the margin (i.e., the high ability "low-skilled" workers) find it worthwhile to acquire middle skill and $\tilde{\theta}_{ML}$ jumps up (top right). This reaction puts a downward pressure on the relative wages of the middle-skilled workers, which in turn pushes the high ability "middle-skilled" workers to acquire high skill, so that $\tilde{\theta}_{HM}$ also jumps up (top middle).

Similar to the reasoning in the first exercise, the decrease in $\lambda_{ML}$ implies a more crowded routine stage labour market in period 2, which immediately discourages product innovation (middle left).

As $\lambda_{ML}$ continues to fall, it promotes a more skilled work force over time, so that the rate of product innovation starts to rise, and the demand for both high- and middle-skilled workers also keep increasing. As long as the demand for skills increases faster than the supply of skills, the relative wage of both high- and middle-skilled workers keep increasing, until they reach their new BGP levels respectively. In summary, a declining process innovation which transmits jobs/tasks from middle-skilled to low-skilled occupations *alone*, cannot generate labour market polarization that we observed.

# 6 Conclusion

Process innovation alters job content which expands the capability of lower skilled workers. In the meantime, scarce higher skilled workers are released and can undertake newer and more demanding jobs. My model captures both the initial discovery of new technologies and the subsequent diffusion of knowledge from the skill frontier to the rest of the labour force. In this diffusion process, the understanding of new technologies becomes further improved and the implementation becomes routinized. At the same time, when skill acquisition is more costly for lower ability workers than for higher ability workers, an increase in skill premium induce more high ability individuals to acquire skills and vice versa. I embed these two mechanisms into an endogenous growth model to study jointly the job market polarization and the slow down in labour productivity growth. I calibrate the model to the European labour market and find that the data supports the model's predictions. I also find that a decline in the rate of process innovation between high-skilled and middle-skilled occupations is essential to understand a polarizing labour market and a slowing down in labour productivity growth.

One limitation of my framework is that labour skill is only one dimensional. It could be helpful to develop a richer framework with more than one type of skills, for example, one could consider general skill versus occupation specific skills. Moreover, as the model is quite stylized, there is no efficiency or intensive margin improvements for workers. A richer model along this dimension may also be better suited for a more detailed empirical investigation.

Figure 4: A negative and permanent shock on $\lambda_{HM}$ (from 2000 to 2014 level)

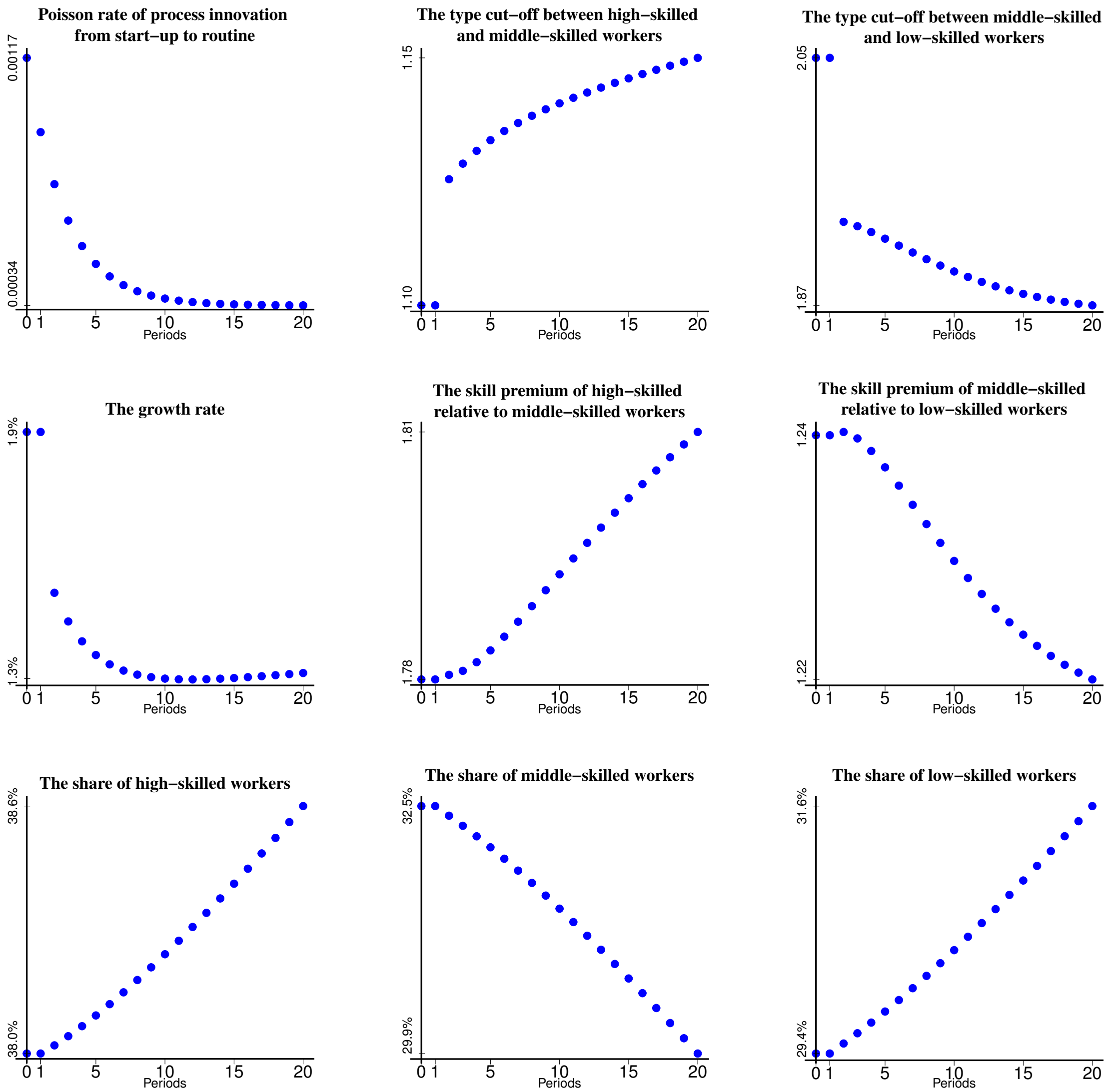
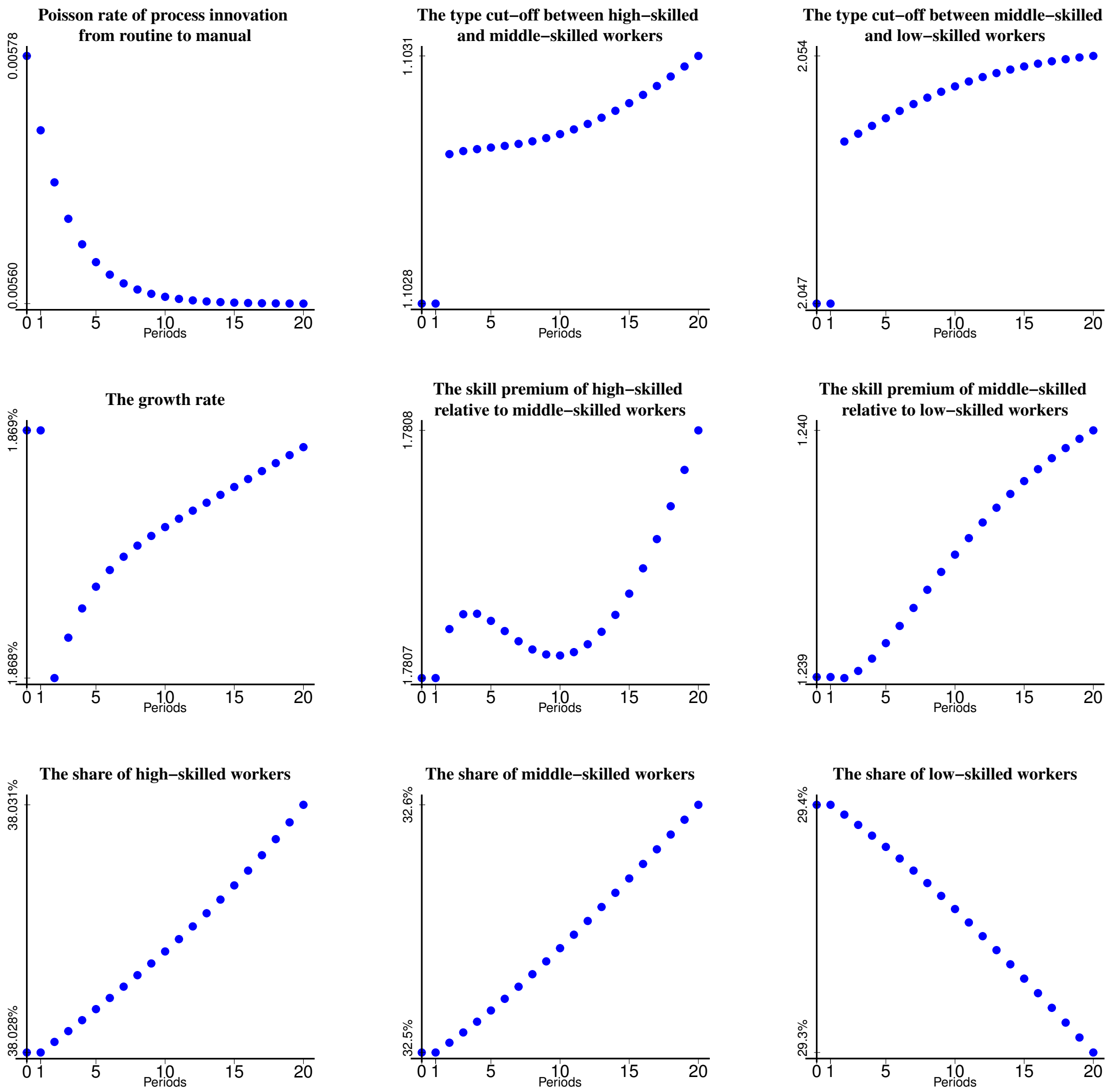


Figure 5: A negative and permanent shock on $\lambda_{ML}$ (from 2000 to 2014 level)



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# A Data Appendix

Table 4 provides detailed information for Figure 2 in Section 2.

Table 4: Proportion of Firms reporting at least one process innovation

| industry | 2000 | 2004 | 2006 | 2008 | 2010 | 2012 | 2014 | 2016 |
|---|---|---|---|---|---|---|---|---|
| C | 21.42% | 17.51% | 17.30% | 19.38% | 16.34% | 13.96% | 14.52% | 23.46% |
| D | 25.50% | 26.68% | 26.33% | 28.57% | 27.06% | 24.90% | 25.32% | 30.12% |
| E | 21.94% | 25.69% | 24.42% | 25.43% | 21.98% | 18.80% | 19.57% | 22.29% |
| F | N/A | 15.52% | 12.86% | 13.62% | 10.84% | 8.41% | 10.40% | 14.21% |
| G | 14.39% | 20.39% | 19.35% | 16.78% | 15.80% | 12.24% | 13.69% | 17.04% |
| H | N/A | 14.63% | 12.53% | 11.38% | 8.82% | 5.40% | 6.72% | 11.05% |
| I | 14.32% | 18.45% | 16.62% | 22.30% | 19.25% | 17.77% | 18.78% | 21.97% |
| J | 31.12% | 29.32% | 26.69% | 25.91% | 27.51% | 22.15% | 23.47% | 27.42% |
| K | 28.96% | 22.66% | 19.52% | 17.95% | 19.25% | 14.61% | 15.15% | 19.75% |
| Average | 22.52% | 21.21% | 19.51% | 20.15% | 18.54% | 15.36% | 16.40% | 20.81% |

Data source: The Community Innovation Survey

Note (1): the industry coding is C - Mining and quarrying, D - Manufacturing, E - Electricity, gas, and water supply, F - Construction, G - Wholesale and retail trade; repair of motor vehicles, motorcycles and personal and household goods, H - Hotels and restaurants, I - Transport, storage and communication, J - Financial intermediation, K - Real estate, renting and business activities.

Note (2): In 2008, the "Statistical Classification of Economic Activities in the European Community" (i.e., the industrial classification) switched from Rev 1.1 to Rev 2. When consolidating the data set, I covert the Rev 2 activities to their Rev 1.1 equivalents based on the code book.

# B Model Specifications and Equilibrium Conditions

In this section, I specify the agents' optimization problems and derive key equations in the main text. I also lay out the equilibrium conditions for the continuous time version of the model.

**Final good producers' problem**

$$\max_{x_{s,j}(t)} PY(t) - \int_{j \in N(t)} p_{s,j}(t) x_{s,j}(t), s \in \{H, M, L\}, \tag{37}$$

where the price of the final good is normalized to 1 (i.e., $P = 1$). F.O.C. $x_{s,j}(t)$, I get

$$MC_{s,j}(t) = MP_{s,j}(t), \tag{38}$$

$$p_{s,j}(t) = N(t)^{\frac{2\alpha-1}{\alpha}} \frac{1}{\alpha} \left[ \int_{j\in N_L(t)} x_{L,j}^{\alpha}(t)dj + \int_{j\in N_M(t)} x_{M,j}^{\alpha}(t)dj + \int_{j\in N_H(t)} x_{H,j}^{\alpha}(t)dj \right]^{\frac{1}{\alpha}-1} \alpha x_{s,j}^{\alpha-1}(t), \tag{39}$$

which can be simplified and get Equations 2.

**Intermediate producers' problem**
For an intermediate producer $j$ at stage $s \in \{H, M, L\}$

$$\max_{p_{s,j}(t)} p_{s,j}(t)x_{s,j}(t) - w_s(t)x_{s,j}(t) \tag{40}$$

F.O.C. $p_{s,j}(t)$, I get

$$x_{s,j}(t) + \left(p_{s,j}(t) - w_s(t)\right) N(t)^{\frac{2\alpha-1}{1-\alpha}} Y(t) \frac{1}{\alpha-1} p_{s,j}(t)^{\frac{1}{\alpha-1}-1} = 0, \tag{41}$$

which can be simplified and get Equation 6.

**New workers' problem**
New worker with type $\theta$ seeks to maximize his/her contribution to the household's wealth, by acquiring the most rewarding skill level.

$$\max \left\{ \int_t^{t+T} w_H(v)e^{-\bar{r}(t,v)(v-t)}dv - \mu\theta N(t), \right.$$
$$\int_t^{t+T} w_M(v)e^{-\bar{r}(t,v)(v-t)}dv - \theta N(t),$$
$$\left. \int_t^{t+T} w_L(v)e^{-\bar{r}(t,v)(v-t)}dv \right\},$$

which yields the two no arbitrage conditions 9 and 10.

**Households' problem**

$$\max U(c(t)) = \int_t^{\infty} \log(c(v))e^{-\rho(v-t)}dv,$$
$$\text{s.t. } \dot{a}(t) = a(t)r(t) + w(t) - c(t),$$

where $c(t)$ denotes the final good consumption at time $t$, $\rho$ denotes the subjective discount factor, $a(t)$ denotes the asset level, $w(t)$ denotes the household level wage income from its workers, and $r(t)$ denotes the interest rate. The household's optimization behavior yields the Euler equation 14.

**Equilibrium conditions for the continuous time version of the model**

The dynamic equilibrium of the the continuous time version of the model can be characterized by the following system of five differential equations (with some initial conditions and transversality conditions), in the $(\chi_H, \chi_M, \tau_H, \tau_M, \tilde{c})$ space,

$$\frac{\dot{\chi_H}(t)}{\chi_H(t)} = \frac{g(t)}{\chi_H(t)} - \lambda_{HM} - g(t), \tag{42}$$

$$\frac{\dot{\chi_M}(t)}{\chi_M(t)} = \lambda_{HM}\frac{\chi_H(t)}{\chi_M(t)} - \lambda_{ML} - g(t), \tag{43}$$

$$\frac{\dot{\tau_H}(t)}{\tau_H(t)} = \frac{\tilde{\theta}_{HM}(t) - \tilde{\theta}_{HM}(t-T)}{\int_{t-T}^{t}\left[\tilde{\theta}_{HM}(v) - \underline{\theta}\right]dv}, \tag{44}$$

$$\frac{\dot{\tau_M}(t)}{\tau_M(t)} = \frac{\tilde{\theta}_{ML}(t) - \tilde{\theta}_{HM}(t) - [\tilde{\theta}_{ML}(t-T) - \tilde{\theta}_{HM}(t-T)]}{\int_{t-T}^{t}\left[\tilde{\theta}_{ML}(v) - \tilde{\theta}_{HM}(v)\right]dv}, \text{ and} \tag{45}$$

$$\frac{\dot{\tilde{c}}(t)}{\tilde{c}(t)} = r(t) - \rho - g(t), \tag{46}$$

where $\tilde{c}(t) \equiv C(t)/N(t)$ and $\{g(t), \tilde{\theta}_{HM}(t), \tilde{\theta}_{ML}(t), r(t)\}$ solves the following four equations,

$$\tilde{\theta}_{HM}(t) = \frac{\int_t^{t+T}[w_H(v) - w_M(v)]e^{-\bar{r}(t,v)(v-t)}dv}{N(t)\cdot(\mu-1)}, \tag{47}$$

$$\tilde{\theta}_{ML}(t) = \frac{\int_t^{t+T}[w_M(v) - w_L(v)]e^{-\bar{r}(t,v)(v-t)}dv}{N(t)}, \tag{48}$$

$$V_H(t) = \frac{1}{\eta}, \text{ and} \tag{49}$$

$$Y(t) = C(t) + I_{PD}(t) + I_L(t). \tag{50}$$

# C The Derivation and Existence of Balanced Growth Path

In this section, I first derive the equations for the BGP equilibrium, and then provide a simple proof of its existence.

Recall that in the BGP, aggregate output, aggregate investment, and aggregate consumption, measures of intermediate producers at each production stages, employment and wages of different skilled levels and thereby prices of intermediate goods, all grow at the same constant rate $\hat{g}$. In other words, $\dot{\chi_H}(t) = \dot{\chi_M}(t) = \dot{\tau_H}(t) = \dot{\tau_M}(t) = \dot{\hat{c}}(t) = 0$ along the BGP.

First of all, in the BGP equilibrium, shares of the start-up, the routine, and the manual stage intermediate producers are constant over time:

$$N_H(t) = \frac{\hat{g}}{(\hat{g} + \lambda_{HM})} \cdot N(t) \equiv \hat{\chi}_H \cdot N(t), \tag{51}$$

$$N_M(t) = \frac{\hat{g}\lambda_{HM}}{(\hat{g} + \lambda_{ML})(\hat{g} + \lambda_{HM})} \cdot N(t) \equiv \hat{\chi}_M \cdot N(t), \text{ and} \tag{52}$$

$$N_L(t) = \frac{\lambda_{HM}\lambda_{ML}}{(\hat{g} + \lambda_{ML})(\hat{g} + \lambda_{HM})} \cdot N(t) \equiv \hat{\chi}_L \cdot N(t). \tag{53}$$

Similarly, shares of workers with high, middle, and low skill are also constant along the BGP, as the worker type cut-off levels $\{\hat{\theta}_{HM}, \hat{\theta}_{ML}\}$ are constant. In other words, the skill composition does not change along the BGP:

$$\hat{\zeta}_H = \frac{\hat{\theta}_{HM} - \underline{\theta}}{\bar{\theta} - \underline{\theta}}, \hat{\zeta}_M = \frac{\hat{\theta}_{ML} - \hat{\theta}_{HM}}{\bar{\theta} - \underline{\theta}}, \text{and } \hat{\zeta}_L = \frac{\bar{\theta} - \hat{\theta}_{ML}}{\bar{\theta} - \underline{\theta}};$$
$$\hat{\tau}_H = \hat{\zeta}_H T, \hat{\tau}_M = \hat{\zeta}_M T, \text{and } \hat{\tau}_L = \hat{\zeta}_L T.$$

As a result, profits for intermediate producers at each stage are constant.

$$\hat{\pi}_H = (1-\alpha)\hat{y}^{1-\alpha}\left(\frac{\hat{\chi}_H}{\hat{\tau}_H}\right)^{-\alpha}\mathcal{L}, \tag{54}$$

$$\hat{\pi}_M = (1-\alpha)\hat{y}^{1-\alpha}\left(\frac{\hat{\chi}_M}{\hat{\tau}_M}\right)^{-\alpha}\mathcal{L}, \text{and} \tag{55}$$

$$\hat{\pi}_L = (1-\alpha)\hat{y}^{1-\alpha}\left(\frac{\hat{\chi}_L}{\hat{\tau}_L}\right)^{-\alpha}\mathcal{L}, \tag{56}$$

where $\hat{y} = \left[\hat{\chi}_L^{1-\alpha}\hat{\tau}_L^{\alpha} + \hat{\chi}_M^{1-\alpha}\hat{\tau}_M^{\alpha} + \hat{\chi}_H^{1-\alpha}\hat{\tau}_H^{\alpha}\right]^{\frac{1}{\alpha}}$.

Note that, while wages grow along the BGP, the income distribution is constant:

$$\hat{w}_H(t) = \alpha\hat{y}^{1-\alpha}\left(\frac{\hat{\chi}_H}{\hat{\tau}_H}\right)^{1-\alpha} N(t), \tag{57}$$

$$\hat{w}_M(t) = \alpha\hat{y}^{1-\alpha}\left(\frac{\hat{\chi}_M}{\hat{\tau}_M}\right)^{1-\alpha} N(t), \tag{58}$$

$$\hat{w}_L(t) = \alpha\hat{y}^{1-\alpha}\left(\frac{\hat{\chi}_L}{\hat{\tau}_L}\right)^{1-\alpha} N(t), \tag{59}$$

$$\frac{\hat{w}_H(t)}{\hat{w}_M(t)} = \left(\frac{\hat{g} + \lambda_{ML}}{\lambda_{HM}} \Big/ \frac{\hat{\theta}_{HM} - \underline{\theta}}{\hat{\theta}_{ML} - \hat{\theta}_{HM}}\right), \tag{60}$$

$$\frac{\hat{w}_M(t)}{\hat{w}_L(t)} = \left(\frac{\hat{g}}{\lambda_{ML}} \Big/ \frac{\hat{\theta}_{ML} - \hat{\theta}_{HM}}{\bar{\theta} - \hat{\theta}_{ML}}\right). \tag{61}$$

With the help of these derivations, Equation 32 is derived by substituting Equation 29, 30, and 31 into Equation 28, and impose BGP (i.e., setting $\dot{V}_H$, $\dot{V}_M$, and $\dot{V}_L$ equals to zero and etc). Equations 33 and 34 are derived by simplifying Equations 9 and 10 respectively.

**Proof of Proposition 1**

I prove the existence of BGP by deriving $\hat{g}$. Suppose that $\lambda_{HM} \to \infty$ and $\lambda_{ML} \to \infty$, which implies that new intermediate goods would become "manual" as soon as they enter the economy. In this case, $N_H(t) = N_M(t) = 0$, and $N_L(t) = N(t)$. Meanwhile, no worker would choose to acquire high- or middle- skill, and thereby $\tilde{\theta}_{HM} = \tilde{\theta}_{ML} = \underline{\theta}$.

Consequently, $\pi_L = (1-\alpha)\mathcal{L}T$, and $w_L(t) = \alpha N(t)$.

$$V_H = \frac{\pi_L}{\hat{r}} = \frac{1}{\eta}, \tag{62}$$

$$\frac{\pi_L}{\hat{g} + \rho} = \frac{1}{\eta} \tag{63}$$

So that

$$\hat{g} = \eta(1-\alpha)\mathcal{L}T - \rho. \tag{64}$$

# D Equilibrium Conditions for the Discrete Time Version of the Model and the Algorithm to Compute the Transitional Dynamics

In this section, I first present the equilibrium conditions for the discrete time version of the model, which is used in the quantitative exercise. I then provide the detailed algorithm to compute the transitional paths.

The dynamic equilibrium of the economy can be characterized by a nine-equation system as the following for the discrete time version of the model, which is analogous to Equations 42 to 50 for the continuous time version:

$$\chi_{H,t} = -(\lambda_{HM} + g_{t-1} - 1)\chi_{H,t-1} + g_{t-1}, \tag{65}$$

$$\chi_{M,t} = -(\lambda_{ML} + g_{t-1} - 1)\chi_{M,t-1} + \lambda_{HM}\chi_{H,t-1}, \tag{66}$$

$$\tau_{H,t} - \tau_{H,t-1} = \frac{1}{T}\left(\frac{\tilde{\theta}_{HM,t} - \underline{\theta}}{\bar{\theta} - \underline{\theta}} - \frac{\tilde{\theta}_{HM,t-T} - \underline{\theta}}{\bar{\theta} - \underline{\theta}}\right), \tag{67}$$

$$\tau_{M,t} - \tau_{M,t-1} = \frac{1}{T}\left(\frac{\tilde{\theta}_{ML,t} - \tilde{\theta}_{HM,t}}{\bar{\theta} - \underline{\theta}} - \frac{\tilde{\theta}_{ML,t-T} - \tilde{\theta}_{HM,t-T}}{\bar{\theta} - \underline{\theta}}\right), \tag{68}$$

$$\frac{\hat{c}_{t+1} - \hat{c}_t}{\hat{c}_t} = r_t - \rho - g_t, \tag{69}$$

$$\tilde{\theta}_{HM,t}(\mu - 1) = \frac{\hat{w}_{H,t}}{1 + r_t} + \sum_{v=t+1}^{t+T-1} \frac{\hat{w}_{H,v}\Pi_{\omega=t}^{v-1}(1 + g_\omega)}{\Pi_{\omega=t}^{v}(1 + r_\omega)} - \frac{\hat{w}_{M,t}}{1 + r_t} - \sum_{v=t+1}^{t+T-1} \frac{\hat{w}_{M,v}\Pi_{\omega=t}^{v-1}(1 + g_\omega)}{\Pi_{\omega=t}^{v}(1 + r_\omega)}, \tag{70}$$

$$\tilde{\theta}_{ML,t} = \frac{\hat{w}_{M,t}}{1 + r_t} + \sum_{v=t+1}^{t+T-1} \frac{\hat{w}_{M,v}\Pi_{\omega=t}^{v-1}(1 + g_\omega)}{\Pi_{\omega=t}^{v}(1 + r_\omega)} - \frac{\hat{w}_{L,t}}{1 + r_t} - \sum_{v=t+1}^{t+T-1} \frac{\hat{w}_{L,v}\Pi_{\omega=t}^{v-1}(1 + g_\omega)}{\Pi_{\omega=t}^{v}(1 + r_\omega)}, \tag{71}$$

$$V_t = \sum_{v=t}^{\infty} \frac{\Phi_{H,v}\pi_{H,v} + \Phi_{M,v}\pi_{M,v} + \Phi_{L,v}\pi_{L,v}}{\Pi_{\omega=t}^{v}(1 + r_\omega)} = \frac{1}{\eta}, \tag{72}$$

$$g_t = \eta\left(y_t\mathcal{L}T - \hat{c}_t - \frac{I_{L,t}}{N_t}\right), \tag{73}$$

where $\hat{c}_t \equiv C_t/N_t$ and $\frac{I_{L,t}}{N_t} = \frac{1}{2}\mathcal{L}[\tau_{H,t}\tilde{\theta}_{HM,t}\mu+(\tilde{\theta}_{HM,t}+\tilde{\theta}_{ML,t})\tau_{M,z}]$. Note also that $\{\Phi_{H,v}, \Phi_{M,v}, \Phi_{L,v}\}_{v=1}^{\infty}$ are the transition probabilities of process innovations for intermediate producers.

The algorithm to calculate the transitional paths in Section 5 is similar to a typical shooting algorithm and can be carried out with the following steps:

- Step 1. Calculate the BGP outcomes, for both before and after the shock, using the discrete time BGP equations similar to Equations 32, 33, 34, and 35.
- Step 2. Assume after $\mathcal{T}$ periods the economy would arrive at the new BGP.
- Step 3. Guess policies for the $\mathcal{T}$ transitional periods
  - $\{g_t, \theta_{HM,t}, \theta_{ML,t}\}_{t=0}^{t=\mathcal{T}}$.
- Step 4. Calculate the state variables $\{\chi_{H,t}, \chi_{M,t}, \tau_{H,t}, \tau_{M,t}\}_{t=0}^{t=\mathcal{T}}$ for the $\mathcal{T}$ periods, using Equations 65, 66, 67, and 68, respectively,
  - then calculate $\{y_t\}_{t=0}^{t=\mathcal{T}}$,
  - then use the resource constraint (Equation 73) to compute $\{\hat{c}_t\}_{t=0}^{t=\mathcal{T}}$,
  - then use the Euler equation (Equation 69) to compute $\{r_t\}_{t=0}^{t=\mathcal{T}}$.
- Step 5. Calculate optimal policies based on these guesses for $\mathcal{T}$ periods, by solving the nine-equation system above starting from period 2 and moving forwards,
  - and get $\{g_t^*, \theta_{HM,t}^*, \theta_{ML,t}^*\}_{t=0}^{t=\mathcal{T}}$.
  - Note that $\chi_{H,t}$ and $\chi_{M,t}$ are predetermined, so in each iteration I solve for these two variables first (using Equations 65 and 66), and then solve for the remaining seven variables simultaneously.
- Step 6. Compare $\{g_t^*, \theta_{HM,t}^*, \theta_{ML,t}^*\}_{t=0}^{t=\mathcal{T}}$ with the guess in Step 3. If they are close enough, then stop.
  - If not, then update the guesses and go back to Step 4.

One aspect of the algorithm that worth mentioning is that, during the transition, $\lambda_{HM,t}$ and $\lambda_{ML,t}$ will keep decreasing, so that every entering cohort of firms face a different transition matrix from each other, until $\lambda_{HM,t}$ and $\lambda_{ML,t}$ arrives at their new "steady state" value, respectively.